CALCULATION OF γ-RAY DOSE RATE FROM AIRBORNE AND DEPOSITED ACTIVITY

CERC

In this document 'ADMS' refers to ADMS 5.2.

Introduction

The γ-dose module calculates the γ-dose rate on the ground at all the user-defined output points. Individual contributions from each isotope in the plume are output. Contributions from airborne and deposited material are calculated separately.

Calculation of gamma dose due to airborne material

The general expression for the effective flux of gamma rays at a point $\boldsymbol{r}$ from a source of energy E dispersed in air is:

$$\Phi(r,E) = \iiint \frac{f(E)C(r)B(E,\mu|r-r'|)\exp(-\mu|r-r'|)}{4\pi(r-r')^2} d^3r' \tag{1}$$

where C is the concentration in $Bq\ m^{-3}$ of the isotope being considered, $f(E)$ the branching ratio to the specified energy, B the build up factor, and μ the linear attenuation coefficient. The build-up factor B is calculated from Berger's analytic expression

$$B(E,\mu r) = 1 + a(E)\mu r \exp(b(E)\mu r) \tag{2}$$

μ and the coefficients $a(E)$ and $b(E)$ are obtained from tabulated data (shown in Table 1). The total dose rate is obtained by summing over all energy groups. The effective dose rate on the body is obtained by multiplying the flux at energy E_i by an absorption coefficient μ_{ai}, given by

$$\mu_{ai} = \frac{\mu_i}{1 + \frac{a_i}{(1-b_i)^2}} \tag{3}$$

then multiplying by a conversion coefficient C_{bi}, and then summing over all energy so that the dose rate D is

$$D = \sum_i C_{bi} \mu_{ai} E_i \Phi_i \tag{4}$$

Dose rates appropriate to different organs of the body can be calculated by using absorption coefficients appropriate to the particular organ being considered. The dose rates used in ADMS, which are listed in Table 1, are those suitable for calculating a total effective body dose.

Since the integrals cannot in practice be evaluated precisely, approximate methods are used. We employ spherical polar coordinates (r, θ, ϕ) centred on the receptor point, so that (1) becomes

$$\Phi = \frac{1}{4\pi} \iiint f(E) C(r, \theta, \phi) B(E, \mu r) e^{-\mu r} dr \sin\theta d\theta d\phi \tag{5}$$

We consider a situation such that in a wind-aligned Cartesian system the release, dose and emission points are $(0,0,H_s)$, $(X_D, 0,0)$ and $(\varepsilon, \eta, \zeta)$ respectively (i.e. the dose point at which the gamma dose is to be calculated is under the plume centreline). The polar axis is taken to be in the ζ direction and to pass through the receptor point. We then have

$$\begin{aligned} \varepsilon &= X_D + r\sin\theta\cos\phi \\ \eta &= r\sin\theta\sin\phi \\ \zeta &= r\cos\theta \end{aligned}$$

and the integration range is

$$\begin{aligned} &0 \le r \le R \\ &0 \le \theta \le \pi/2 \\ &0 \le \phi \le 2\pi \end{aligned}$$

Note that as the integrand (5) contains the factor $e^{-\mu r}$ it decreases rapidly as r increases, so that a finite value R can be used.

We use a Gaussian quadrature method to evaluate (5). The Gauss-Legendre N-point quadrature formula is

$$\int_{x_1}^{x_2} f(x) dx = \frac{x_1 - x_2}{2} \sum_{i=1}^{N} w_i f(x_i) \tag{6}$$

with abscissae x_i and weights w_i. Therefore, (5) becomes

$$\begin{aligned} \Phi &= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi/2} \int_0^R f(E) C(r, \theta, \phi) B(E, \mu r) \exp(-\mu r) dr \sin\theta d\theta d\phi \\ &= \frac{1}{4\pi} \left(\frac{R}{2} \times \frac{\pi}{4} \times \pi \right) \sum_{k=1}^{N} w_k \sum_{j=1}^{N} w_j \sum_{i=1}^{N} w_i f(E) C(r_i, \theta_j, \phi_k) B(E, \mu r) \exp(-\mu r_i) \sin\theta_j \end{aligned} \tag{7}$$

where r_i and w_i are the abscissae and weights of the Gauss-Legendre N-points quadrature in the range of integration $[0, R]$, similarly, θ_j and w_j, ϕ_k and w_k are the abscissae and weights in the range of integration $[0, \pi/2]$ and $[0, 2\pi]$ respectively.

R is chosen to be the radius at which the gamma rays are attenuated to 1% of their original strength. This distance varies with energy level and is given by

$$e^{-\mu_i R} = 0.01 \tag{8}$$

In order that the radius of integration is not too small, a minimum value of $R = 30$ m is taken. We have tested the sensitivity of the numerical calculation to the value of N in different meteorological conditions. The value used is $N = 15$ which gives a balance between accuracy and computational time.

Expression (1), for the effective flux of gamma rays may be simplified in two cases. Firstly, if the plume is narrow and elevated, the gamma rays can be assumed to radiate from a **line source** [2]. It is assumed that the plume is sufficiently narrow and elevated if it satisfies the following conditions:

$$\frac{z_p}{\sqrt{(\sigma_y \sigma_z)}} > 5$$

and

$$\sigma_y < 1 \text{ m and } x > 20d_s \quad \text{or} \quad \sigma_z < 0.05R \text{ and } x > 20d_s$$

where x is the downstream distance from the source and d_s is the source diameter. Equation (1) is then evaluated as a line integral. The conditions ensure that the line source approximation is used when the plume is too shallow or narrow to be resolved accurately by the Gaussian quadrature integration method, and is not used when modelling ground-based plumes.

Secondly, if the radioactive cloud is large compared to the mean free path of the γ-rays, and satisfies the following criteria

$$\begin{aligned}
&\mu(1-b)\sigma_z > 3 \\
&\mu(1-b)\sigma_z^2 > 3z_p \\
&\mu(1-b)h > 2 \\
&\mu(1-b)\sigma_y > 3 \\
&\mu(1-b)\sigma_y{}^2 > 3y_d
\end{aligned}$$

then the **semi-infinite cloud approximation** [1] is used. σ_z and σ_y are the vertical and horizontal plume spread parameters, z_p is the plume height, h the boundary layer height, and y_d is the perpendicular distance of the output point from the plume centreline. This gives

$$\Phi(r,E) = \frac{f(E)C(r)}{2\mu_a} \tag{9}$$

Calculation of gamma dose due to deposited material

The NRPB [3] suggests that the gamma dose rate due to deposited material G_{dep} can be calculated in a similar way to the contribution from airborne material, as follows:

$$G_{dep} = \sum_i C_{bi}\,\mu_{ai} E_i F_i \tag{10}$$

where

$$F = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\infty} \frac{f(E) \times D \times B(E,\mu r)e^{-\mu r}}{r}\, dr d\theta \tag{11}$$

where D is the activity deposit per unit area.

In ADMS, for continuous plume releases, it is the deposition *flux* that is calculated, i.e. the deposit per unit area per unit time. The above calculation has been implemented in the model with the deposition flux substituted for the total deposit D. Hence the model outputs the *change in* G_{dep} *per unit time*, (G_{dep}', say) in units of Sv/s/s.

The gamma dose rate G_{dep} in Sv/s can be calculated from the model results with a little post processing. At time T after the start of the release, the gamma dose rate G_{dep} will be given by

$$G_{dep} = \int_{t=0}^{T} G_{dep}'\, dt \tag{12}$$

So, for example, if the release starts at a particular time and continues indefinitely at a constant rate, then G_{dep} at time T will be given by

$$G_{dep} = G_{dep}' \times T$$

Note that strictly speaking the calculation should take into account the decay of material after it has been deposited. However, in the ADMS calculations this effect has been ignored for simplicity. This approach is satisfactory if the isotopes of interest have half-lives much longer than the period of interest.

Calculation of gamma dose away from the plume centreline

The points at which the gamma dose is to be calculated may lie off the plume centreline. The receptor points (x, y) are transformed to wind-aligned coordinates (x_d, y_d) within the gamma dose module.

The three-dimensional integration method used in most cases to calculate the gamma dose needs no alteration for points away from the centreline. For the semi-infinite cloud approximation the off-centreline gamma dose is calculated from the centreline dose by multiplication by an appropriate factor:

$$dose(x_d, y_d) = dose(x_d, 0) \times \exp\left(\frac{-{y_d}^2}{2{\sigma_y}^2}\right) \tag{13}$$

where σ_y at x_d is calculated by interpolation.

In the narrow plume approximation the crosswind distance y_d is incorporated into the distance of the point from the plume centreline.

Calculation of long-term average gamma dose

The long-term average gamma dose is calculated using the same method as that used to calculate long-term average concentrations (described in P07/04). If the input wind direction data are binned into large sectors, for each line of met data the wind direction ϕ is resolved into n_{wind} equally spaced directions ϕ_i within the sector (n_{wind} is 5). Each of the wind directions is assumed to be equally likely and is assigned the frequency fr/n_{wind}, where fr is the frequency of the met line. For wind data in small sectors, or not in sectors, $n_{wind} = 1$. The gamma dose at each output point is calculated for each wind direction. The long term average gamma dose at each point (x_d, y_d) is then given by

$$dose(x_d, y_d) = (1/ftotal) \times \sum_{metdata} \sum_{i=1}^{n_{wind}} dose_i(x_d, y_d) \times fr/n_{wind} \qquad (14)$$

where $dose_i(x_d, y_d)$ is the concentration when the wind direction is ϕ_i and $ftotal$ is the total frequency, i.e. the sum of the frequencies of all the lines of met data.

References

[1] Corbett, J.O. The calculation of external gamma-ray dose from airborne and deposited radionuclides in the environmental code NECTAR. CEGB Report RD/B/5201/N82, February 1982.

[2] Jones, J.A. and Charles, D. AD-MARC: The atmospheric dispersion module in the methodology for assessing the radiological consequences of accidental releases. NRPB Report M72, September 1982.

[3] ESCLOUD: A computer program to calculate the air concentration, deposition rate and external dose rate from a continuous discharge of radioactive material to atmosphere. NRPB Report NRPB-R101, J.A. Jones, 1980.

Table 1: Values of μ, $a(E)$, $b(E)$, $\mu_a E$ and C_b used in the model for different energy values E

$E(MeV)$	$\mu(m^{-1})$	a	b	$\mu_a E(Gym^{+2})$	$C_b(Sv/Gy)$
0.01	0.623	0.025	–0.0464	7.43×10^{-16}	0.00296
0.015	0.187	0.0947	–0.0484	3.12×10^{-16}	0.0183
0.02	0.0893	0.2652	–0.0463	1.68×10^{-16}	0.0543
0.03	0.0411	1.055	–0.0192	0.721×10^{-16}	0.191
0.05	0.0253	3.498	0.0729	0.323×10^{-16}	0.557
0.065	0.0226	4.209	0.1169	0.278×10^{-16}	0.63
0.1	0.0195	4.033	0.1653	0.371×10^{-16}	0.765
0.2	0.0159	2.678	0.1678	0.856×10^{-16}	0.703
0.5	0.0112	1.748	0.1014	2.38×10^{-16}	0.689
1.0	0.00821	1.269	0.0559	4.47×10^{-16}	0.732
1.5	0.00668	1.040	0.0338	6.12×10^{-16}	0.765
2.0	0.00574	0.891	0.0215	7.50×10^{-16}	0.791
4.0	0.00398	0.5879	0.0022	12.0×10^{-16}	0.850
10	2.65×10^{-3}	0.3113	–0.0194	23.1×10^{-16}	0.935