AIRCRAFT SOURCES

CERC

In this document 'ADMS' refers to ADMS-Airport 4.1.

---

1. Introduction

Aircraft can emit a significant magnitude of emissions at low levels. It is therefore important to model their dispersion accurately in order to obtain valid predictions of pollutant concentrations in and around airports.

Aircraft engine exhaust emissions usually have high exit velocities relative to ambient wind speeds. Locally, for example during take-off when the aircraft is close to the ground, the source properties of the exhaust jets are important for accurate dispersion modelling. Therefore, ADMS includes an **aircraft** source type that takes into account the effects of buoyancy, momentum and aircraft motion on the dispersion of exhaust material.

The more traditional method of modelling emissions from aircraft as volume sources is still valid, and indeed recommended, for other parts of the LTO cycle, such as climb out, when the local source properties are less important.

2. Treatment of Aircraft Sources in ADMS

Each aircraft source represents a section of the aircraft's trajectory, e.g. take-off ground roll or initial climb. An aircraft source is defined as a collection of sub-sources, where each sub-source represents a trajectory for a particular aircraft category. Each sub-source is represented in ADMS as a series of continuous jet source releases, equally spaced along the aircraft's trajectory. Each engine on the aircraft is represented individually, up to a limit of four engines.

For general information about jets and directional releases in ADMS, please see Technical Specification paper P11/02.

The user is required to enter a number of engine parameters for each aircraft category, which are required to enable ADMS to model the aircraft jet releases; these are exhaust exit velocity V_e, exhaust temperature T and exhaust diameter D. These can be calculated from typically-available data as described below.

3. Aircraft Source jet component parameters

3.1 Background information

Commercial passenger aircraft tend to use high-bypass ratio turbofan jet engines, such as the one sketched in **Figure 1**, for their superior fuel efficiency. The term ‘bypass’ refers to the air that passes through the fan at the front of the engine, but ‘bypasses’ the engine core itself, gaining momentum from the fan only. The ‘core’ flow is the air that feeds the engine core, which is first compressed to very high pressures in the ‘compressor’ and then mixed with fuel and burnt in the ‘fuel burner’. This hot gas is used to drive the turbine that powers the compressor and fan shaft and then the nozzle converts the high pressure to high velocity, generating thrust. The ratio of bypass flow to core flow is termed the engine’s ‘bypass ratio’.

In the derivations below, the exhaust flow is assumed to be the mixture of the hot core and cool bypass flows, and this mixture is assumed to be uniform in temperature and velocity.

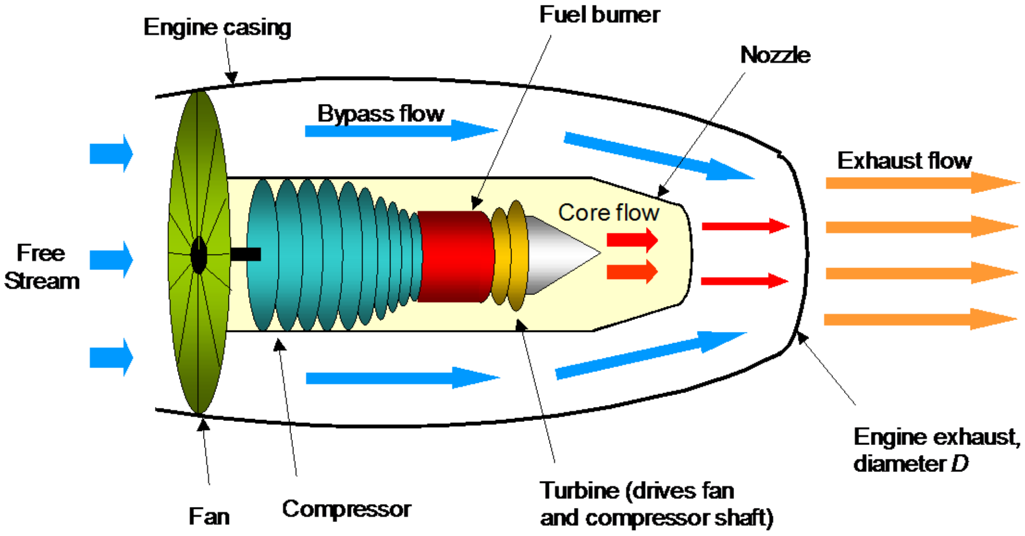


Figure 1 – Schematic diagram of the airflow through a high-bypass ratio turbofan jet engine

3.2 Derivation of the exit velocity

The aircraft thrust is equal to the rate of change of momentum of air passing through the aircraft engine:

$$Thrust(N) = \dot{m}(V_e - V_0)$$

where $\dot{m}$ is the total mass flow rate, V is the flow velocity and the subscripts *0* and *e* refer to values at entry and exit from the engine respectively, relative to the moving aircraft; V_0 is the aircraft speed.

Rearranging this gives

$$V_e = \frac{Thrust}{\dot{m}} + V_0 .$$

The exit velocity relative to a stationary observer on the ground is therefore

$$V_e - V_0 = \frac{Thrust}{\dot{m}}.$$

Engine thrust (N) and mass flow rate (kg/s) data during take-off are both available in the spreadsheets accompanying the textbook 'Civil Jet Aircraft Design' [1].

3.3 Derivation of the source temperature T

The amount of energy per second produced by the combustion of aviation fuel in the engine, Q (J/s), can be derived from the fuel flow rate during take-off, $\dot{f}$ (kg/s) [2] and the known energy of combustion of aviation fuel (43.5 MJ/kg from [3]):

$$\dot{Q} = \dot{f} \times 43.5 \times 10^6 \text{ J/s}$$

The thermal efficiency η of an aircraft engine is the ratio of the total work done by the engine to the total energy obtained from fuel combustion. This can be estimated as

$$\eta = \frac{\text{Power output} + \text{Kinetic energy gained by the air}}{\text{Energy from fuel combustion}}$$

Thermal efficiency can also be written

$$\eta = \frac{Thrust \times V_0 + \frac{1}{2}\dot{m}\left(V_e - V_0\right)^2}{\dot{Q}}$$

The amount of heat available to raise the temperature of the exhaust is $(1-\eta)\dot{Q}$.

Once we have estimated the thermal efficiency η, it is possible to estimate the temperature of the exhaust gases:

$$T_{exhaust} = T_{ambient} + \frac{(1-\eta)\dot{Q}}{\dot{m}c_p}$$

where c_p is the specific heat capacity of dry air at constant pressure, which is 1000.4 J/kg/K.

3.4 Derivation of the source diameter D

No data are typically available for the engine exhaust diameter *D*, although it is typically about equal to the fan diameter for turbofan engines with separate jets, and equal to three quarters of the fan diameter for mixed flow engines[1] (**Figure 1**). However, the diameter can be derived from the Ideal Gas Equation:

$$P = \frac{\rho R_* (T + 273.15)}{M},$$

where *P* is the pressure (taken to be 101300 Pa), M is the molar mass of air (0.02896 kg/mol), R_* is the Universal Gas Constant (8.314 J/(Kmol)), *T* is the source temperature and ρ is the density, defined as:

---

[1] Paul Madden, Rolls Royce Group PLC, private communication, 9 July 2004

$$\rho = \text{density} = \frac{\dot{m}}{\frac{\pi D^2}{4} \times V_e}.$$

Therefore:

$$D(m) = \sqrt{(T + 273.15) \times \frac{\dot{m}}{V_e} \times \frac{8.314}{0.02896 \times 101300} \times \frac{4}{\pi}}.$$

4. Effect of the moving aircraft on jet plume dispersion

4.1 Introduction

All discussions in this section refer to the components of the motion that are in the horizontal plane.

For a jet source such as an aircraft engine moving through the air with a speed V_A, the dispersion of material is different to the case where the jet source is stationary. In a frame of reference moving with the source, the aircraft speed acts like an extra component of the wind speed.

In the jet source dispersion calculations in ADMS, we consider two frames of reference: one moving with the source at a speed V_A, and the other stationary. The plume rise part of the dispersion calculations is carried out in the moving frame, where the ambient wind speed is modified to include the additional component induced by the movement of the source (resulting in an 'effective' wind speed). In the moving frame, the exit velocity is also increased, because the inflow to the engine is increased, and the engine must maintain the same amount of acceleration of the airflow in order to maintain thrust. The rest of the dispersion calculations (calculation of plume spread and concentration) are done in the stationary frame as they are more related to the ambient meteorological conditions.

4.2 Effective wind speed U' and direction φ'

The additional wind speed V_A experienced by the engine in the moving frame affects the effective wind direction as well as the effective wind speed (see **Figure 2**). In terms of the velocity (i.e. speed vectors), if the aircraft moves with speed $V_A = |\boldsymbol{V_A}|$ in a direction $-\boldsymbol{V_A}$, then the jet release direction is $\boldsymbol{V_A}$. Therefore:

- for a zero ambient wind, the effective wind speed in the moving frame of reference is $\boldsymbol{V_A}$, and
- for a non-zero ambient wind, the effective wind speed in the moving frame of reference is $\boldsymbol{V_A} + \boldsymbol{U}$, where $\boldsymbol{U}$ is the ambient wind speed vector.

If the ambient wind direction is φ (measured clockwise from north in the usual way) and the jet release direction is α (measured anticlockwise from east), the resultant 'effective wind' speed U' and direction φ' are described as follows:

$$U' = \sqrt{(U_E')^2 + (U_N')^2}$$
$$\phi' = \tfrac{3\pi}{2} - \arctan\left(U_N' \big/ U_E' \right),$$

where U_E' and U_N' are the components of U' in the eastwards and northwards directions respectively:

$$\begin{aligned} U_E' &= U \times \cos\left(\tfrac{3\pi}{2} - \phi\right) + V_A \times \cos\alpha \\ U_N' &= U \times \sin\left(\tfrac{3\pi}{2} - \phi\right) + V_A \times \sin\alpha \end{aligned},$$

and $U = |\boldsymbol{U}|$.

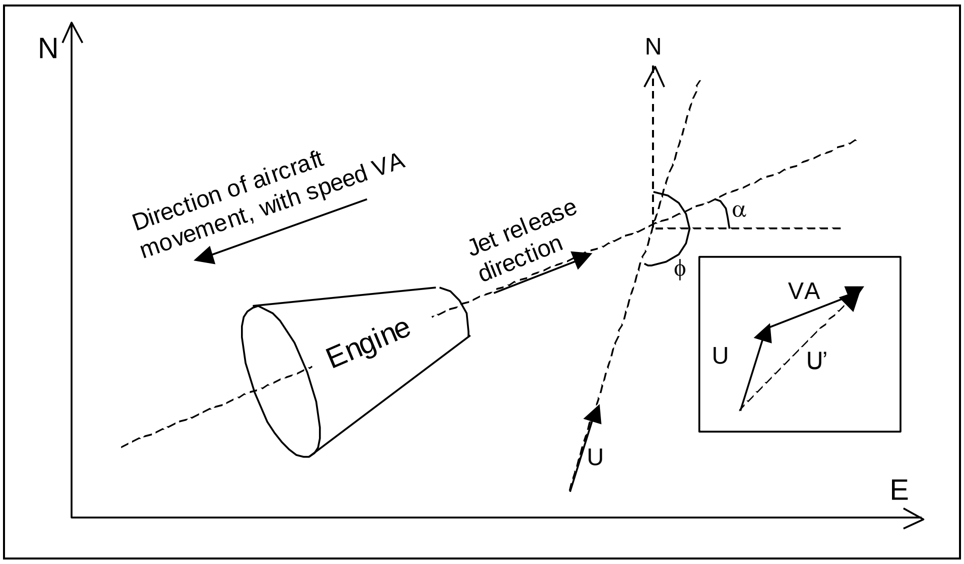


Figure 2 – Schematic representation of the effective wind direction in relation to the ambient wind direction and the aircraft travel direction

4.3 Transformation of the plume trajectory between moving and stationary frames of reference

The plume trajectory in the stationary frame is represented by the coordinates *(X,Y)* where *X* is the distance from the source in the ambient wind direction and *Y* is the distance from the source in a direction perpendicular to the ambient wind direction.

The plume trajectory in the moving frame is represented by *(X',Y')* where *X'* is the distance from the source in the effective wind direction and *Y'* is the distance from the source in a direction perpendicular to the effective wind direction.

The plume rise calculations are carried out in the moving frame, where the plume trajectory is represented by *(X',Y')*; the rest of the dispersion calculations are carried out in the stationary frame, where the plume trajectory is represented by *(X,Y)*.

The transformation of the plume trajectory between the two frames of reference has two parts:

- Rotation of the plume trajectory to account for the difference between the ambient wind direction and the effective wind direction, and
- Translation of the plume trajectory to account for the distance travelled by the moving frame.

Rotation

For the rotation part of the transformation between stationary and moving frames of reference, if γ is the angle between the ambient wind direction and the effective wind direction (measured anti-clockwise from the ambient wind direction to the effective wind direction) then:

$\gamma = \varphi - \varphi'$,

$X' = X\cos\gamma + Y\sin\gamma$, and

$Y' = -X\sin\gamma + Y\cos\gamma$.

Translation

Please refer to **Figure 3** for a schematic of the translation in the simplest case, where the aircraft direction of travel is opposite to the wind direction. In time *dt*, the plume travels from $x=x_0$ a distance $U_P \times dt$ in the stationary frame of reference, to the point represented by the black block on the figure, where U_P is the plume speed, i.e:

$$x = x_0 + U_P \times dt$$

In the moving frame, the plume speed is U_P', so the position x' in the moving frame after time *dt* is represented by:

$$x' = x_0 + U_P' \times dt$$

U_P and U_P' are related by:

$$U_P' = U_P + V_A$$

Solving the above three equations leads to:

$$x' = x + V_A \times dt.$$

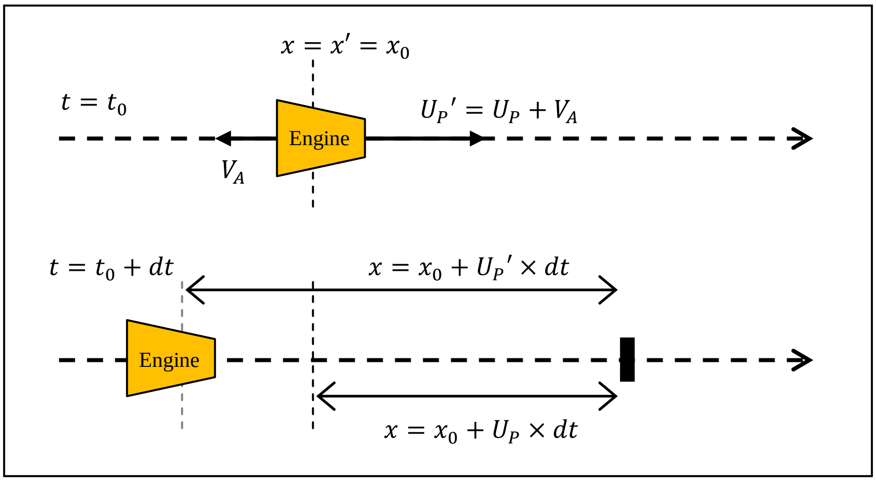


Figure 3 – Schematic of the translation between moving and stationary frames of reference for an aircraft engine travelling with speed V_A, where the ambient wind and the aircraft speed are in opposite directions

In the more general case, where the ambient wind and the aircraft are not travelling in opposite directions (refer to **Figure 4**): if β is the angle from the aircraft direction of travel to the effective wind direction (measured anticlockwise), then:

$$\beta = \left(\tfrac{3\pi}{2} - \varphi'\right) - \alpha$$
$$X' = X + V_A t\cos\beta$$
$$Y' = Y - V_A t\sin\beta$$

where *(X,Y)* has already been rotated to the effective wind direction and *t* is the elapsed time since the plume left the source.

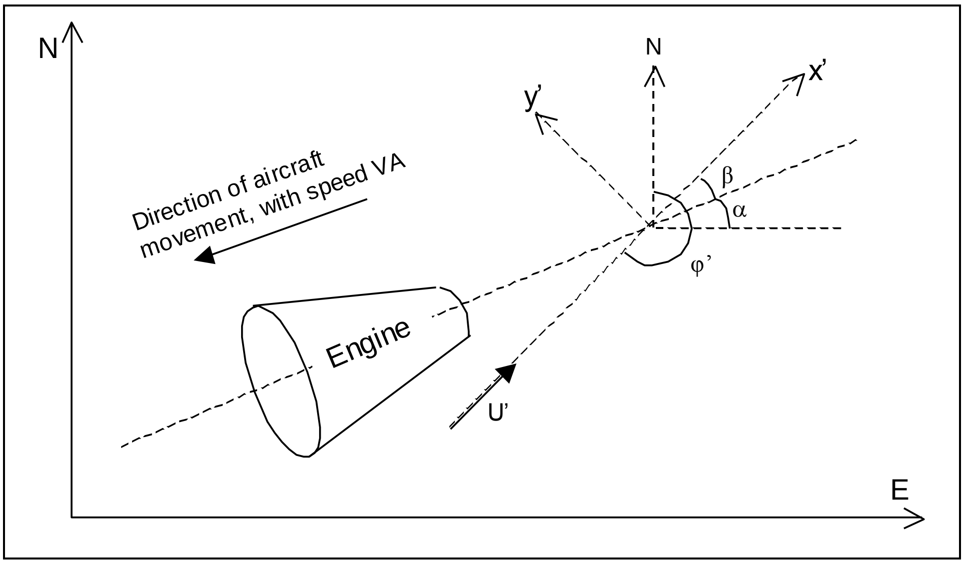


Figure 4 – Schematic representation of the relationship between the movement of the aircraft and the rotated frame of reference aligned with the effective wind direction

4.4 *Transformation of the plume velocity between moving and stationary frames of reference*

The plume velocity as calculated by the plume rise calculations is, of course, the plume velocity in the frame of reference moving with the source, and therefore it will be notated here as (U_P',V_P'). The plume velocity is used in various other parts of the dispersion calculations, where it must be the plume velocity in the stationary frame of reference, i.e. (U_P,V_P). The transformation from (U_P',V_P') to (U_P,V_P) is very similar to the inverse of the transformation described above for the plume trajectory (outlined above) i.e. firstly, the translation:

$$\begin{aligned} U_P'' &= U_P' - V_A \cos\beta \\ V_P'' &= V_P' + V_A \sin\beta \end{aligned} ,$$

and, secondly, the rotation:

$$\begin{aligned} U_P &= U_P''\cos\gamma - V_P''\sin\gamma \\ V_P &= U_P''\sin\gamma + V_P''\cos\gamma \end{aligned} .$$

4.5 *Jet exit velocity in the moving frame of reference*

The exit velocity V_e input by the user for a particular aircraft category is considered to be the engine exhaust exit velocity in the stationary frame of reference, in zero wind conditions. This is due to the assumptions made during the derivation of the exit velocity from the thrust value, i.e. zero inflow speed. The actual exit velocity in the stationary frame is therefore the input value plus the component of the wind in the jet release direction. In the moving frame, the exit velocity V_e' is the value in the stationary frame plus the aircraft speed.

$$\begin{aligned} \beta_0 &= (\tfrac{3\pi}{2} - \varphi) - \alpha \\ V_e' &= V_e + U\cos\beta_0 + V_A \end{aligned} .$$

5. Representation of the aircraft sub-source as a collection of jet sources

5.1 *Introduction*

As mentioned in Section 2, an aircraft sub-source is represented in ADMS as a series of continuous jet source releases. This section describes how the jet sources are distributed along the aircraft sub-source, and outlines how the total emission rate Q (g/s) is apportioned between the jet sources.

5.2 *Jet source locations*

The number of jets used to model a particular sub-source, NT, is a user-defined parameter. In addition, for a particular sub-source, the user specifies:

$(X0, Y0)$ – the (x,y) coordinates of the aircraft starting position (m)

$(X1, Y1)$ – the (x,y) coordinates of the aircraft finishing position (m)

The distance travelled by the aircraft, L, is calculated by the model as:

$$L = \sqrt{(X1 - X0)^2 + (Y1 - Y0)^2}\ .$$

Distance L is split into NT sections of equal length. The jet sources are placed longitudinally at the centre of each section, and transversely at locations corresponding to the engines. For example, **Figure 5** is a diagram showing the flight path from $(X0, Y0)$ to $(X1, Y1)$ for an aircraft with two wing-mounted engines, and NT=9. The direction of travel is from left to right, and red circles show the locations of the jets.

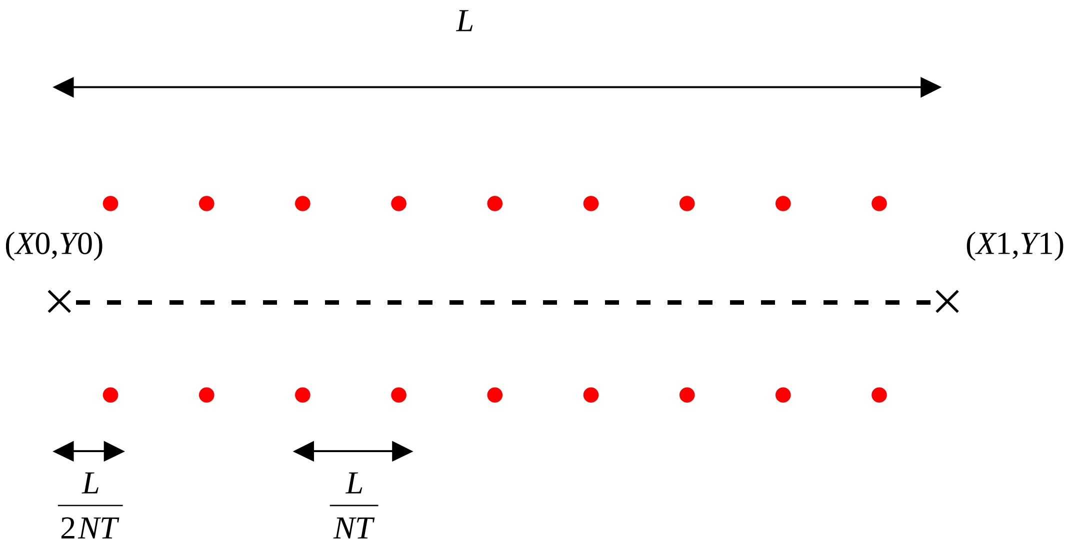


Figure 5 – Location of jets (red dots) relative to aircraft sub-source location (dotted line)

5.3 Apportionment of emissions (for the constant acceleration case)

Figure 6 shows example velocity curves for accelerating (full line) and decelerating (dashed line) aircraft. The emission rate, Q (g/s), is apportioned according to the time taken for the source to travel between jet locations. This means that if a plane is accelerating, for example during take-off or climb out, the emission rates for individual jets decrease along the source; conversely when a plane is decelerating, for example during the landing roll, the emission rates for individual jets increase along the source.

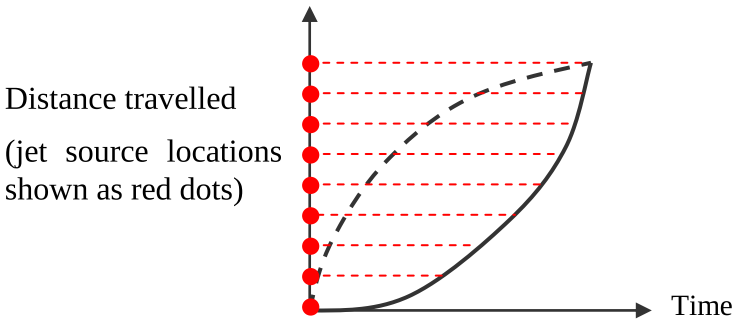


Figure 6 – Example velocity curves, with the full line indicating an accelerating trajectory, and the dashed line indicating a decelerating trajectory

5.4 Apportionment of emissions (for the non-constant acceleration case)

Unless otherwise specified, aircraft acceleration and pollutant emissions are assumed to be constant and the pollutant emission rates for the individual jet sources are calculated as described in section 5.3. However, for take-off in particular this is not always the case. To address this, the user can specify that the aircraft exhibits non-constant acceleration, and distribute the emission of pollutants variably along the take-off trajectory. The equations used to model the change in speed and emissions along the trajectory are as follows.

5.4.1 Speed development during take-off

The development of speed during take-off is assumed to be governed by the equation:

$$V = V_{to}\left(A_S\left(\frac{t}{T_{to}}\right)^2 + B_S\frac{t}{T_{to}} + C_S\right)\left(D_S.\tanh\left(E_S\frac{t}{T_{to}} + F_S\right) + G_S\right)$$

V – speed (m/s)

V_{to} – take-off speed (m/s)

t – time (s)

T_{to} – take-off time (s)

A_S, B_S, C_S, D_S, E_S, F_S and G_S – speed development coefficients

5.4.2 Emission development during take-off

Development of emissions can be defined separately for each aircraft category and applied to specific pollutants. The development of emissions during take-off is assumed to be governed by the equation:

$$Qfactor = \left(A_E \left(\frac{t}{T_{to}} \right)^3 + B_E \left(\frac{t}{T_{to}} \right)^2 + C_E \frac{t}{T_{to}} + D_E \right) \left(E_E . \tanh\left(F_E \frac{t}{T_{to}} + G_E \right) + H_E \right)$$

Qfactor – emissions factor

t – time (s)

T_{to} – take-off time (s)

A_E, B_E, C_E, D_E, E_E, F_E, G_E, H_E – normalised emission development coefficients

The area under the curve (t/T_{to}) against Qfactor must be equal to 1.

6. Intelligent gridding for Aircraft Sources

6.1 Introduction

The concentration gradients across an aircraft's trajectory are significant; therefore ADMS includes a gridding tool called 'Intelligent gridding' to increase output grid resolution around aircraft sources.

Note that intelligent gridding also applies to road and line sources; this section deals only with aircraft sources.

6.2 Placement of intelligent grid points

Each sub-source has start and end coordinates. In order to define the geometry of an aircraft source from the sub-sources it contains, the model calculates the coordinates of the smallest 'enclosing rectangle' containing all sub-sources. The width of each sub-source is defined as the distance between its outermost engines. For the purposes of defining the enclosing rectangle, each sub-source is extended by 10 times its width behind the start, in order to resolve the concentration distribution from the jets at the beginning of the sub-source (recalling that the jets point backwards relative to the aircraft movement).

Start and end coordinates and a width define the enclosing rectangle for each sub-source. ADMS positions additional output points in and around this enclosing rectangle, up to a maximum of 2000 points. Points are added in sets of 8 where each set of 8 points lies on a line perpendicular to, and centred on, the axis of the enclosing rectangle. The spacing between each set of 8 points is equal to the maximum of the width of the enclosing rectangle and *MinSpacing*, where

$$MinSpacing = \max\left(0.005 \times GridExtent, S_{\min}\right).$$

Here

$$GridExtent = \sqrt{(DX \times DY)}\ ,$$

DX and DY are the extent of the output grid (in metres) in the x and y directions respectively and

$$N_{rwy} = total\ number\ of\ aircraft\ sources$$

$$L_{rwy} = total\ aircraft\ source\ length$$

$$S_{\min} = \frac{8 \times L_{rwy}}{2000 - 8 \times N_{rwy}}.$$

This process of imposing a lower limit on the along-source spacing between sets of points ensures that the resolution is no higher than necessary, and that the available points are distributed evenly.

7. Estimating engine exhaust parameters

7.1 Introduction

Engine exhaust parameters required for modelling aircraft sources using the ADMS jet model may not always be available from the manufacturer. For this reason, a method has been developed of estimating engine exhaust temperature and engine exhaust velocity from the engine bypass ratio (BPR), a more widely available metric.

7.2 Estimating parameters

In lieu of manufacturer data, aircraft engine exhaust parameters can be estimated from the bypass ratio of the engine. As part of the DfT Air Quality Studies for Heathrow (**DfT** (2007)) manufacturer data were provided for 10 airframe-engine combinations in 4 modes of operation; take-off, initial climb, landing and taxiing. The BPR of the engines was taken from the ICAO Engine Emissions Databank (**ICAO** (2005)) for the engines specified.

The engine BPRs, exhaust velocity and temperature conditions were plotted, see **Figure 7** - Relationships between aircraft engine bypass ratio (BPR) and (a) engine exhaust velocity and (b) engine exhaust temperature**Figure 7**. There is a clear linear relationship between BPR and exhaust conditions. Engine exhaust velocity and temperature can be estimated using the following equations and data given in **Table 1**. This method allows engine exhaust parameters to be estimated for jet engines larger than 26.7 kN thrust contained in the ICAO Engine Emissions Databank.

$$\text{Engine Exhaust Velocity} = m_v.BPR + c_v$$

$$\text{Engine Exhaust Temperature} = m_T.BPR + c_T$$

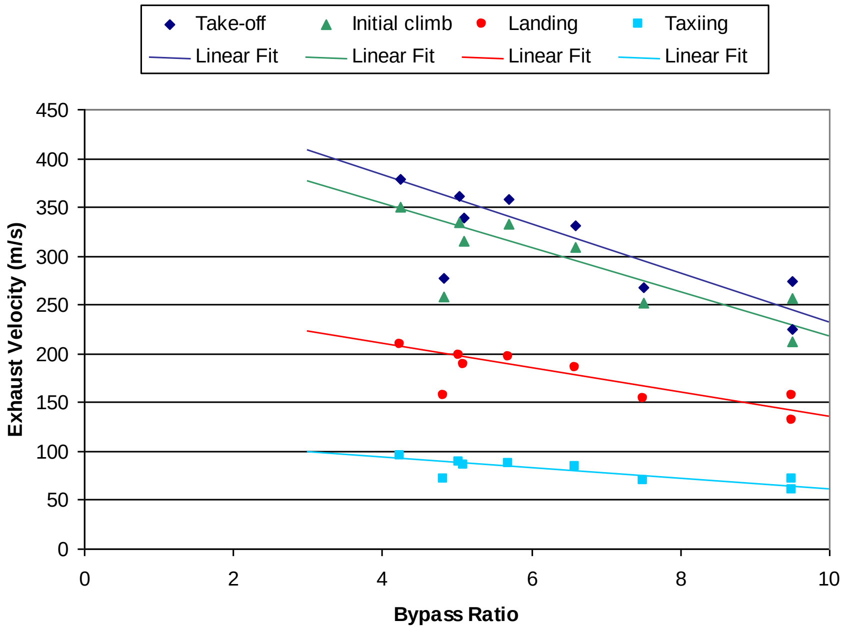
BPR – bypass ratio

m_v, c_v – exhaust temperature coefficients

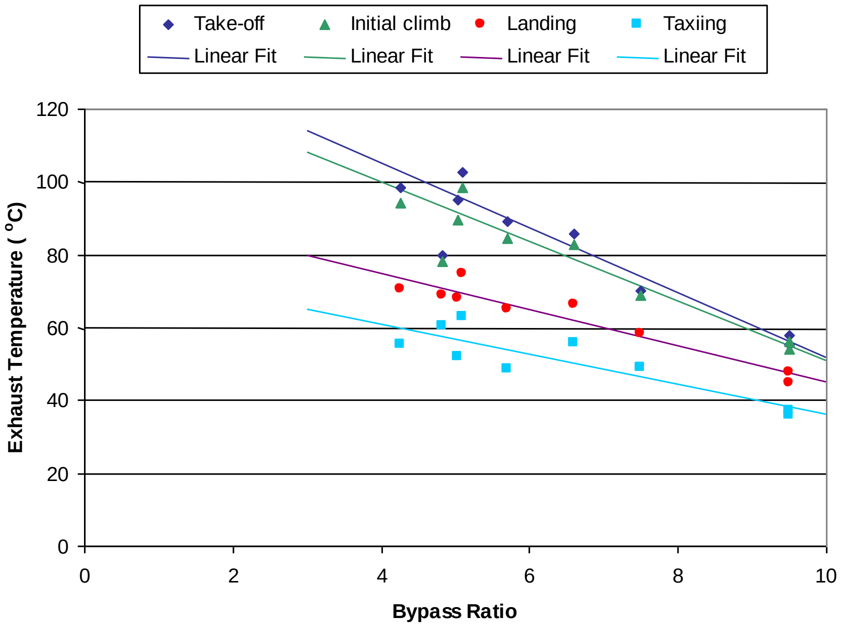
m_T, c_T – exhaust temperature coefficients

Aircraft Mode	**Thrust Setting**	**m_v**	**c_v**	**m_T**	**c_T**
Take-off	100%	-25.27	485	-8.86	141
Initial climb	85%	-22.65	446	-8.17	133
Landing	30%	-12.44	260	-4.98	95
Taxiing	7%	-5.52	117	-4.10	77

Table 1 - Coefficients for estimating engine exhaust parameters from bypass ratio



(a)



(b)

Figure 7 - Relationships between aircraft engine bypass ratio (BPR) and (a) engine exhaust velocity and (b) engine exhaust temperature

8. References

[1] Spreadsheets published online to accompany the textbook ‘Civil Jet Aircraft Design’: http://www.bh.com/companions/034074152X/appendices/data-b/default.htm

[2] ICAO ‘Aircraft Engine Exhaust Emissions Databank’: http://www.qinetiq.com/aircraft.html

[3] "Aircraft Fuel." Ed. Sybil P. Parker. *McGraw-Hill Encyclopedia of Science and Technology*. 8th ed. N.p.: R.R. Donnelly and Sons Company, The Lakeside P, 1997.